LESSON 5. 6a

Inverse of a function

Today you will:

- Explore inverses of functions.
- Find and verify inverses of nonlinear functions.
- Practice using English to describe math processes and equations

Prior Vocabulary:

- input / domain
- output / range
- inverse operations
- reflection
- line of reflection

Given $f(x) = x + 2$

1. Evaluate $f(x)$ when $x = 3$
   - $f(3) = 3 + 2 = 5$

2. Evaluate $f(x)$ when $y = 7$
   - $y = f(x)$
   - $y = x + 2$
   - $7 = x + 2$ … solve for x
   - $x = 7 - 2 = 5$

…when finding x given y, first solve $f(x)$ for x
…rewrite the function as $x =$

- $f(x) = x + 2$
- $y = x + 2$ … write as $y =$
- $y - 2 = x$ … solve for x
- $x = y - 2$ … rearrange as $x =$

Let $f(x) = 2x + 3$.

a. Solve $y = f(x)$ for x.

b. Find the input when the output is -7.

SOLUTION

a. $y = 2x + 3$ — Set y equal to $f(x)$.

$y - 3 = 2x$ — Subtract 3 from each side.

$\frac{y-3}{2} = x$ — Divide each side by 2.

b. Find the input when $y = -7$.

$x = \frac{-7 - 3}{2}$ — Substitute -7 for y.

$= \frac{-10}{2}$ — Subtract.

$= -5$ — Divide.

Check

$f(-5) = 2(-5) + 3$

$= -10 + 3$

$= -7$ ✓

So, the input is -5 when the output is -7.

When we solved $f(x)$ for x what we did was find the ***inverse of*** $\boldsymbol{f(x)}$

Definition: Inverse of a function

- The inverse of a function is the "reverse" of that function
- What you get when you swap the inputs and outputs of a given function
- …when you swap the domain and range of a given function…
- …when you swap the x and y ***VALUES*** of a given function

Example: $f(x) = 2x + 3$ original function $g(x) = \dfrac{x - 3}{2}$ inverse function

Seeing the inverse through tables of data…

Original function: $f(x) = 2x + 3$

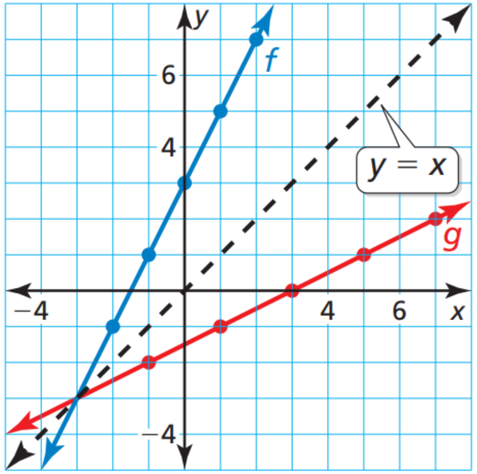
x	−2	−1	0	1	2
y	−1	1	3	5	7

Inverse function: $g(x) = \dfrac{x - 3}{2}$

x	−1	1	3	5	7
y	−2	−1	0	1	2

x and y values swapped…

Seeing the inverse through graphing…



Notice the graph of the inverse function is the reflection of the original around the line $y = x$

Process to find the inverse of a function:

1. Rewrite the function as $y =$

2. In this equation, swap x and y

3. Solve for y

4. The new function from step 3 is the inverse of the original

Find the inverse of $f(x) = 3x - 1$.

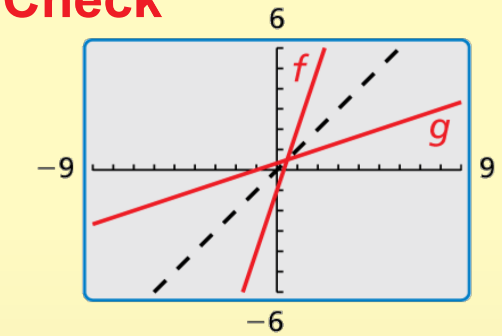
SOLUTION

Set y equal to $f(x)$. Switch the roles of x and y and solve for y.

$y = 3x - 1$	Set y equal to $f(x)$.
$x = 3y - 1$	Switch x and y.
$x + 1 = 3y$	Add 1 to each side.
$\dfrac{x+1}{3} = y$	Divide each side by 3.

The inverse of f is $g(x) = \dfrac{x+1}{3}$, or $g(x) = \dfrac{1}{3}x + \dfrac{1}{3}$.

Check



The graph of g appears to be a reflection of the graph of f in the line $y = x$. ✓

Process to find the inverse of a function:

1. Rewrite the function as $y =$
2. In this equation, swap x and y
3. Solve for y
4. The new function from step 3 is the inverse of the original

Find the inverse of $f(x) = x^2$, $x \geq 0$. Then graph the function and its inverse.

SOLUTION

$f(x) = x^2$	Write the original function.
$y = x^2$	Set y equal to $f(x)$.
$x = y^2$	Switch x and y.
$\pm\sqrt{x} = y$	Take square root of each side.

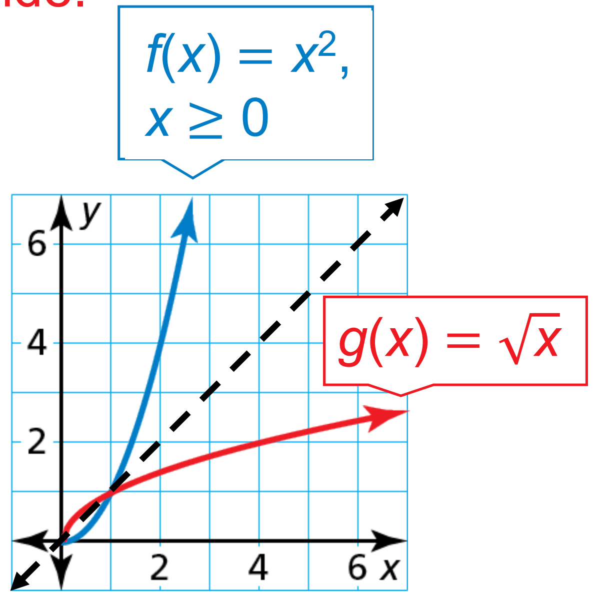
Now consider any possible restrictions…

The domain of f is restricted to nonnegative values of x. So, the range of the inverse must also be restricted to nonnegative values.

So, the inverse of f is $g(x) = \sqrt{x}$.

STUDY TIP

If the domain of f were restricted to $x \leq 0$, then the inverse would be $g(x) = -\sqrt{x}$.



How restrictions on $f(x)$ affect its inverse function…

- Because we get the inverse of $f(x)$ by swapping its domain (input or x values) with its range (output or y values)…
  - The domain restrictions of $f(x)$ apply to the range of its inverse
  - The range restrictions of $f(x)$ apply to the domain of its inverse

$f(x)$

Domain restrictions

Range restrictions

$g(x)$

Domain restrictions
- same as range restrictions of $f(x)$

Range restrictions
- Same as domain restrictions of $f(x)$

How do we tell if an equation is a function?

- Vertical line test

Is there a way by looking at a function if its inverse will be a function?

- The inverse is found by swapping domain and range (x values with y values)…
- …we are "flipping" the x and y's…
- …what would we get if we "flipped" a vertical line?
- …a ***horizontal line***!

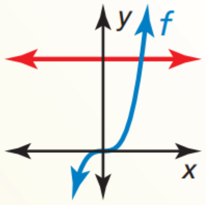
The inverse of a function is itself a function if the original functions passes the "horizontal line test"

Core Concept

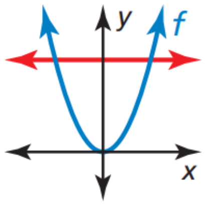
Horizontal Line Test

The inverse of a function *f* is also a function if and only if no horizontal line intersects the graph of *f* more than once.

Inverse is a function



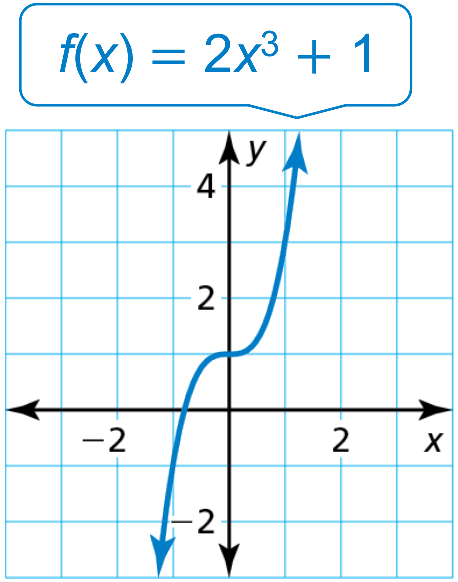
Inverse is not a function



Consider the function $f(x) = 2x^3 + 1$. Determine whether the inverse of f is a function. Then find the inverse.

SOLUTION

Graph the function f. Notice that no horizontal line intersects the graph more than once. So, the inverse of f is a function. Find the inverse.



$$y = 2x^3 + 1 \qquad \text{Set } y \text{ equal to } f(x).$$

$$x = 2y^3 + 1 \qquad \text{Switch } x \text{ and } y.$$

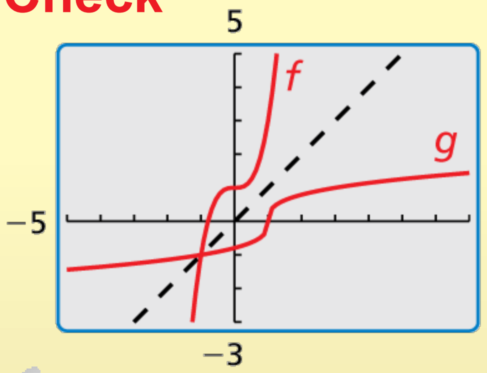
$$x - 1 = 2y^3 \qquad \text{Subtract 1 from each side.}$$

$$\frac{x-1}{2} = y^3 \qquad \text{Divide each side by 2.}$$

$$\sqrt[3]{\frac{x-1}{2}} = y \qquad \text{Take cube root of each side.}$$

$$\frac{\sqrt[3]{4x-4}}{2} = y \qquad \text{Rationalize the denominator.}$$

Check



▶ So, the inverse of f is $g(x) = \dfrac{\sqrt[3]{4x-4}}{2}$.

Homework

Pg 281, #5-36